# REPORT
# OF THE
# MATHEMATICS REFORMS COMMITTEE

**UNIVERSITY GRANTS COMMISSION**
**NEW DELHI**
**1966**

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# CONTENTS

*Foreword*

1. Introduction ...

2. Existing Facilities for Teaching Mathematics in our Universities ... ... ... 5

3. Examinations ... ... ... 9

4. Proposed Syllabus in Mathematics ... ... 12

Appendix—Record of Discussions with Academician Prof. A. Alexandrov, Rector, Leningrad University, Regarding Revision of Mathematics Course ... ... ... 23

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# FOREWORD

The University Grants Commission appointed a committee under the chairmanship of Dr. D.C. Pavate to suggest measures for the improvement of the standard and quality of mathematics in universities and colleges in the country. The committee has prepared a report which would be of real value and help to all interested in the development of study and research in mathematics. The commission is grateful to the chairman and to the members of the committee for giving their valuable time and attention to the work of the committee and the preparation of the report.

D.S. Kothari
*Chairman*
*University Grants Commission*

New Delhi
November, 1966.

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# 1

# INTRODUCTION

To meet the challenge of the continuous growth of scientific knowledge, the universities have to maintain some balance between the old and the new in their programme of studies. The University Grants Commission realised the great hiatus between the courses of studies in various subjects in our country and those of the advanced countries in the other parts of the world. The U.G.C., therefore, appointed a few years back 'Review Committees' in a number of important subjects in Physical and Biological Sciences, Humanities and Social Sciences, to survey and assess the standards of teaching and research and the facilities available for the purpose. Accordingly, in 1959, a 'Mathematics Review Committee' consisting of some of our distinguished mathematicians was appointed for the purpose. This committee reviewed the position as it existed towards the end of the Second Plan period about the standards of teaching and research in mathematics in this country. One significant suggestion made by this committee was the desirability of reviewing and revising the syllabus once in five years in view of the rapid pace at which knowledge is growing in these days. In pursuance of this suggestion of periodical revision of the mathematical education in this country, the U.G.C. appointed the second Review Committee in October 1964, consisting of the following members :

Dr. D.C. Pavate, ... *Chairman*
Vice-Chancellor,
Karnatak University.

Prof. R.S. Verma,
Head of the Department of
Mathematics and Statistics,
University of Delhi.

Prof. V.S. Huzurbazar,
Head of the Department of
Mathematics and Statistics,
University of Poona.

Prof. N.S. Nagendranath,
Principal, Science College,
Patna University.

Dr. V.S. Patankar, ... *Member—Secretary*
Deputy Secretary,
University Grants Commission,
New Delhi.

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After discussing the problem in a few meetings, the committee drafted an interim report which was sent to all the universities in 1965. The committee held three more meetings in 1966 before drafting the final report.

### The spirit of new mathematics

It has become increasingly clear and imperative that the teaching of mathematics should be so organised that the student gets acquainted with newer concepts as correctly and as early as possible. The classical mathematics used to study isolated and unique models (e. g. the real number system) with extraordinary skill and ingenuity. In such studies it developed several techniques and devices which are very good in themselves. But in any of its results it was difficult to tell how much of it was due to the pattern sustaining the model, and how much was due to the particularity of the situation (e. g. the individual numbers involved in the result). Contrasted with this fact, in modern mathematics we extract abstract structures (i. e. patterns) from particular situations and study these structures. For example, from the particular system of real numbers, we extract the structure of an ordered complete field and establish results in such a structure. The advantage of this type of study is that the knowledge so gained can be applied not only to the particular system of numbers we started with but also to all other systems exhibiting the same pattern. The elegance and the excitement pertaining to the connectibility of diverse domains in one pattern cannot be over-emphasized. The study of structures, apart from unifying seemingly isolated and unconnected domains, leads to a great economy of thought and effort. Let us consider, for example, three classical problems and their isolated solutions :

(1) The problem of finding a root of a polynomial equation like $x^5 + 3x + 4 = 0$ by Newton's method ;

(2) the practical problem of solving a large number of linear equations in a large number of unknowns; and

(3) the problem of solving a system of first order differential equations of the type

$$\frac{dy}{dx} = g\ (x, y) \text{ by various artifices.}$$

It so turns out that the structure of a complete metric space and its mapping into itself is the common pattern of all the three problems. The study of such a common pattern not only unifies the problem but also illuminates a host of other unexpected grounds. A second advantage of such a study is the economy of notation much in the same way as 'vector algebra' simplifies 'classical geometry'.

The main classifications of mathematical structures are (i) the algebraic and (ii) topological structures (including analysis). The language employed in these structures is that of *set theory*, the method of study is the *axiomatic* study and the reasoning made is what we call *deductive reasoning* as opposed to 'intuitive' or 'inductive' reasoning. When we employ deductive reasoning and the axiomatic method we do not deny the usefulness of intuitive thinking in guessing a result or a pattern. We only wish to emphasize that in all processes of thinking, mathematical proof is the *last court* of appeal. The spirit of modern mathematics is thus to encourage logical reasoning, generalization, discovery and development.

## The content of mathematical teaching in universities

Our courses of studies in mathematics had, in the past, some correspondence with those in vogue in Great Britain. We followed more or less what was done in London, Cambridge and other universities in Great Britain. Rarely did we have any impact of other universities in Europe or America on our universities, as far as the content of mathematical teaching was concerned. Books used in Great Britain were taken as models and they had enormous influence on our teaching. Unfortunately, some of them continue to influence us even though they have become out-of-date and have been given up in Great Britain itself. The standard of examinations there was much higher than that obtaining in our country, but the content of mathematical education in India closely resembled that in England. For instance, their VIth form course in mathematics in public and grammar schools approximated to our former Intermediate and in some cases, to the B.A. and B.Sc. courses. About forty or fifty years back, it was easy for a student who had obtained a first class honours in mathematics in an Indian university to obtain a first in the Mathematical Tripos examination (Parts I and II) of the Cambridge University. This is no longer possible as the British universities have changed their courses completely after the second world war and brought them in line with those in the other countries in Europe and America. Formerly the Mathematical Tripos Part I paper contained questions in Algebra, Geometry, Calculus and Trigonometry, which could be solved by our honours students with ease. In applied mathematics only, was their standard of Part I much higher than that of B.A. or B.Sc. honours in Indian universities. In fact, some good honours students did not take the Part I at all; they went straight to Part II. Now Mathematics Tripos Part I papers contain questions on the Theory of Sets, Mathematical Analysis, Infinite Series, Matrices, Groups, Vector Analysis, Vector Space, some of which were not done even for Part II of the Mathematical Tripos before the second world war. Even the General Education Certificate Examination of Great Britain (ordinary level) which is more or less of the standard of our courses in mathematics at the school leaving examination contains questions on *sets* and *matrices.* The truth is that the teachers of mathematics in Great Britain realised some 15 years back that they were lagging behind the other countries in the content of mathematical education and have been now making supreme efforts to improve their standards gradually. Thus today the Indian students are at a great disadvantage in British, German, Russian and American universities as we are still sticking to the old ideas and old books in our universities. For instance, there is hardly any question in the recent Mathematical Tripos papers Parts I and II in Analytical Geometry (Plane or Solid), whereas almost one whole paper in Part II contained questions in that subject some forty years back. Prof. Alexandrov of the Leningrad University, who visited the Indian universities recently, recommended that the teaching of Analytical Geometry should be confined to the bare minimum and that we should switch over to more fundamental material at the earliest possible moment (vide Appendix). In Europe, U.S.A., Russia, Canada and Great Britain, all elementary subjects like Algebra, Geometry and Trigonometry and Calculus are taught with due emphasis on fundamental concepts and logical development. Such a rigour of logic has crept even in schools in many other countries. For instance, Algebra is taught rigorously, i.e. with precise definitions, postulates, and deductive reasoning, right from the beginning in schools. Basic ideas both in Algebra and Geometry require the knowledge of sets and number systems. The American School Mathematics Study Group has produced some interesting books in Algebra and Geometry in advanced 'language and style' which are of great interest to our school and college teachers. Some American mathematicians have also recently held seminars for college and school teachers in our country and this also should contribute to the improvement of the teaching of mathematics in our schools and colleges.

Some universities in India teach Calculus with a fair amount of rigour at the undergraduate stage. This is due to the Cambridge influence which discouraged loose thinking even at the initial stage. Similar efforts are necessary in the teaching of Algebra and Geometry. Here also due emphasis must be laid on essential mathematical concepts and logical reasoning while presenting the usual body of practical and theoretical mathematical knowledge in those subjects. What is done in the upper classes of secondary schools in the U.S.A., Canada and Europe could be done at least in the first two years in our colleges. With this in view we sent an interim report emphasizing the improvement of undergraduate courses. Broadly speaking, we suggested in the interim report that the syllabus in mathematics should be modernised by introducing sets and number systems including basic laws of Algebra, relations, functions and graphs, elements of statistics, and elementary trigonometry. We also suggested the introduction of Abstract Algebra in the higher classes of B.A. or B.Sc. These concepts are of fundamental importance in mathematics.

# 2

# EXISTING FACILITIES FOR TEACHING MATHEMATICS IN OUR UNIVERSITIES

Though the previous review committee made several useful suggestions, they made little impression on the undergraduate teaching of mathematics, although we are aware of some notable exceptions. The reasons for this state of affairs are :

(*a*) The large and steady increase in the number of students in colleges and the lack of corresponding increase in the number of well-qualified teachers.

(*b*) The teachers are, by and large, not inclined to keep themselves abreast of modern developments. Well-qualified teachers are inclined to join the postgraduate departments rather than do undergraduate teaching. The status of a postgraduate teacher is much higher than that of an undergraduate teacher although the work of the latter is in no way less important than that of the former.

(*c*) The questions asked in the competitive examinations of U.P.S.C. continue to be old-fashioned and are not based on modern subjects like Abstract Algebra, and Topology.

(*d*) The engineering colleges attract a large number of potentially good mathematics students and at least in our country waste eventually their talents. This process creates a drain of mathematical talent at the present moment. Suitable incentives such as special scholarships for good mathematics students even at the undergraduate stage and opportunities of employment need to be provided.

(*e*) In some universities the language load is inclined to be heavy at the B.A. or B.Sc. classes. This partly affects the standard of mathematics. While we have no objection to the retention of English or any other European language such as French, German and Russian, the retention of regional language does not serve any useful purpose at this stage.

Somehow a beginning has to be made to modernise the courses of studies in mathematics, but it will not be possible unless we are able to organise seminars every year during the summer vacations, for the education of the teachers. There should be discussion groups and we must indicate to the teachers the lines on which broad advances have been made in mathematics during the last fifty years. Simultaneously, good text-books in mathematics should be got written. Without seminars and without good text-books, no improvement in the mathematical education of undergraduates is possible. It is gratifying to note that some universities and higher secondary school boards are making efforts in this direction and their experiments are well worth encouraging. All the same, great efforts are necessary to modernise the mathematics course as early as possible.

2. Actually much reform is needed in the secondary school mathematics. The introduction to logic could, with advantage, begin from standard VI and be continued through the higher secondary stage. The emphasis should be laid on the discovery and understanding of general principles, but this is a matter which should be seriously considered by the colleges of education and the departments of public instruction. Universities have no say regarding the content of education in the secondary schools except through the colleges of education. The lead in the reform of teaching mathematics in the schools should, therefore, be taken through the colleges of education. The works produced by the American School Mathematics Group will be of great use in this connection.

3. Many universities in India control the examinations of standard XI of the higher secondary schools directly or indirectly by at least laying down the requirements of admission to colleges. Some universities have pre-university classes corresponding to standard XI of the higher secondary schools. It is, therefore, possible to begin the reform in the teaching of mathematics from standard XI or the pre-university class on wards. Hence we suggested in the interim report a few changes in the courses for these classes. Much depends also on the enthusiasm and knowledge of the teachers themselves. Given the will and drive, it is possible to make radical changes in the teaching of mathematics in the colleges and raise the general standard by holding regular annual seminars, etc. and bring our syllabus considerably closer to that followed in advanced countries.

Many educationists hold the view now-a-days that the pre-university course should be of two years' duration and that this could be followed by a three-year degree course. If this is possible, it will make for considerable improvement in the mathematical education in our country. In that case, the first year B.A. and B.Sc. course will automatically be transferred to the higher secondary schools (or pre-university classes) and the undergraduate course can be extended by one year. This will enable us to have a standard comparable to that obtaining in Europe or America to some extent at least. In the present financial stringency, however, it is doubtful whether government will be in a position to find the additional funds required throughout the country. Hence we have made our recommendations in this report on the assumption that the present educational structure, viz. 10-year school education, 1-year pre-university (or 11-year higher secondary school) education, three-year degree course and two-year Master's course, will continue at least for the next five years at the end of which there will be another committee to review the situation.

4. At present, there are three patterns of undergraduate courses of three-years for B.A. and B.Sc. Some universities believe in a three-year general education so that in addition to English they provide for three different courses of the same level. This means that mathematics will have only one-third of the teaching time available. We may put such universities in category I. The standard of mathematical education is bound to be low in such universities at the undergraduate level. The

students in such universities will have to depend only on two-year M.A. and M.Sc. course for their real mathematical education and it seems to us that this period is too short for students of mathematics. The difficulties of these students in tackling research problems or to be effective mathematics teachers in schools and colleges will be also very great indeed.

The second category of universities provide a three-year honours course in mathematics immediately after the pre-university or XIth standard of the higher secondary schools. The only possible criticism against this arrangement is that the students' general education tends to be poor as a result of specialisation at an early stage.

Finally, there are a number of universities which have been taking a sort of middle path between the first and the second categories. We may put such universities in category III. Under this scheme, the students are required to have general education up to the intermediate (or the first year of the B.A. and B.Sc.) stage, followed by a special course in mathematics for two years. The latter course of two years in mathematics closely resembles the honours course in universities in category II. The general standard of mathematical education is more or less the same in universities of categories II and III, although the scheme under the universities of category II has obviously more advantages than in those of category III. For instance, the universities under category III provide general education as well as specialisation. This system would be still more advantageous if it were possible to have a three-year honours or specialised course based on a two-year pre-university course. In the present circumstances, however, we have to see how best we can improve the standard of undergraduate education in mathematics in all the three categories of universities.

5. Mathematical education at least according to the British pattern, usually consists of training in Pure Mathematics and Applied Mathematics. Pure Mathematics provides the basic tools and Applied Mathematics usually consists of Mechanics, Hydrostatics, Hydrodynamics, Electricity and Magnetism, Elasticity and Thermodynamics. These are applications of mathematics to physics. There are also vast applications of mathematics to engineering with which we are not ordinarily concerned in arts and science colleges. Similarly, statistics which has developed into an independent branch of mathematics has many applications to problems both in social and physical sciences. Mathematical treatment of economics has been developing on a large scale during the last thirty years. We have to make provision for the teaching of econometrics at least at the post-graduate level. However, it is not possible to make provision for the teaching of several subjects in Pure and Applied Mathematics in all universities. Some universities, however, should provide for mathematical economics. We can, however, select a few core subjects which could be taught compulsorily at a high level in all the three categories of universities. Now what are these core subjects ? They are : Modern Algebra and Analysis. Not long ago, a mathematician was one who could solve all sorts of difficult examples in Algebra, Trigonometry, Calculus, Pure and Analytical Geometry and Mechanics etc., by learning the tricks of the trade. There was hardly any attention paid to fundamental ideas and logical thinking. Even now, several universities in our

country suffer from these inhibitions in so far as the teaching of mathematics is concerned. The reason for this is the use of out-of-date text-books and the tendency on the part of teachers to stick to what they had learnt in their days. Supreme efforts are, therefore, necessary to remove this inertia and hence fresh thinking is necessary on the content of mathematical syllabus.

Statistics is studied as an independent subject both at the undergraduate and postgraduate levels in several universities. In view of its great importance, it should, in our view, be made a compulsory subject for students taking mathematics as an honours or a major subject at the first degree level. We have given in this report a sort of model syllabus for all the three categories of universities at the undergraduate and postgraduate levels.

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## 3

# EXAMINATIONS

We broadly agree with the previous review committee on this subject. The yrecommended that at least at the M.A./M.Sc. level :—

(i) the teacher teaching the course should be primarily responsible for the assessment,

(ii) the assessment in any particular course should not be based on one single examination at the end of the course,

(iii) a certain percentage of marks (say 25% of the aggregate to start with) should be allotted to periodic tests, consisting of short questions devised to test the students' grasp of the fundamental principles of the course covered during the period,

(iv) at the end of each year the university should conduct an examination on the courses covered during the year; the papers for this examination should be set by internal and external examiners jointly, and

(v) at the end of the full period of the courses for the degree, there should be in addition to the written examination and periodical tests during the year, a *viva voce* examination to guage the students' grasp of the fundamental principles.

The main obstacle in the way of acceptance of these recommendations is the possible partiality of the teacher towards some of his students. The partiality may be due to two reasons : one is that the teachers may have developed special liking for some students and the other is that some pressure may be brought to bear on him by certain outsiders, in respect of some students. The first reason is not a valid argument against a wholly internal examination, for after all, the teacher likes some students mainly because they have some good qualities which bring them to the favourable notice of the teacher. They may answer oral questions in the class or may be quick in imbibing new ideas or may be regular in the submission of their home-work etc. All these are features of good students and there is no harm in giving them their due credit in the final assessment. The second reason is, however, a strong argument against the internal assessment and is not wholly imaginary in the present state of our economic, political and social conditions. Hence we have to move warily in the matter. All the same, periodical tests during the year, carrying about 25% marks will be an excellent corrective to the vagaries of the written examination held at the end of the year. In addition, if a teacher meets each student once a fortnight or a month by special arrangement, he can find out how far his lectures have been grasped by the student, what his weak spots are, and thus can help overcome his individual difficulties and short-comings. This will be in the nature of a 'tutorial' and the teachers can and should meet their students individually or in small groups at least once a month. The marks assigned by this method are most reliable and in course of time, the marks for tutorials and periodical tests may be gradually raised to 50%. Unitary universities like Baroda and Annamalai can abolish the examinations at the end of the year if the tutorials and periodical tests are taken in right earnest by the teachers.

In some universities in India, there is neither a system of periodical (monthly or fortnightly) tests nor the system of annual examination. This, in our opinion, is very detrimental to the interests of students. If there are no tests or examinations for two or three years, there is a natural tendency on the part of the students to take it easy till the last term when they begin to overwork themselves. In our view, there must be a system of annual examination throughout the undergraduate and postgraduate stages and the papers should be set by internal and external examiners jointly. This cannot be avoided in most of our universities at least at present. This is not an unmixed evil ; for it gives an opportunity to the teachers to exchange their views with those of the other universities on the courses of studies, methods of teaching and the maintenance of reasonably uniform standards.

Another evil in the system of our examinations is an endemic tendency to favour the quantitative appoach in [the assessment of student's capacity or intelligence. There is an obsession with actual marks obtained by the candidates in the examinations. Whether it is an award of a scholarship, a fellowship or employment outside, the authorities concerned judge the relative merits of the candidates by the actual marks obtained by them. This is done on the assumption that the marks given by the examiners in a university examination are infallible. With the greatest goodwill on earth, such an assumption can never be correct, and if the same papers are examined by other teachers, the marks obtained may be substantially different from those awarded by the previous examiner even in a subject like mathematics. It would be much safer an truer if the students were declared successful in three or four divisions : 1st class with distinction, 1st class, 2nd class and 3rd class. This is the practice in all the a dvanced countries. The British universities refuse to supply individual marks to anybody. No marks should be supplied to the candidates, passed or failed. This does not mean that the university authorities should not maintain the record of marks obtained by each candidate. This will be necessary for the award of their own medals or prizes or scholarships ; but the information about the exact marks obtained should not be made available to the public for the simple reason that the marks may not indicate the actual ability or knowledge of the candidate concerned.

Another great defect in our present examination system is the great hurry in which the question papers are set. This results in setting stereotyped old questions, often taken from previous papers or the text-books used. In mathematics, the examples set should be simple and straightforward and yet test the thinking power and grasp of the fundamental principles studied by the candidates. If we examine the question papers of our universities, we often find that the questions are difficult and tricky, but can be found in the text-books. There must be a certain amount of freshness in the questions set, which should make the candidates think for themselves. This is not possible if the teachers of colleges and universities try to set papers for several examinations of several universities. It is not possible for a mathematics teacher to set more than two papers for an honours or postgraduate examination. The tendency of giving too many papers (maybe of different universities) to one person year after year, cannot be too strongly deprecated. Younger people should be given a chance of setting papers for an honours or postgraduate examination. Some senior people seem to make a monopoly of this business and that should be checked in the interest of good education. We, therefore, recommend that at least in mathematics :

(i) nobody be a paper-setter for any examination for more than two consecutive years ;

(ii) the examples be fresh, i.e., should not have been taken bodily from the previous years' papers or standard text-books, as far as possible, and the number of questions to be set should not exceed the number of questions to be solved by 25% in the first two years and 50% at the B.A. and B.Sc. and postgraduate examinations ;

(iii) the paper-setters be given at least six months in which to set the papers and supply the answers ;

(iv) the book-work or theory portion should not carry more than 40% marks in an examination up to the first degree examination;

(v) universities encourage all colleges under them to have periodical tests which may carry up to 25% of the total marks to start with, the exact percentage depending on the sincerity and honesty of the colleges and the correlation between these marks and the marks obtained at the annual examinations by the candidates ; and

(vi) the practice of supplying to the candidates the actual marks obtained by them at any examination be done away with, the results being declared only in four classes, keeping the actual marks obtained by them strictly confidential.

# 4

# PROPOSED SYLLABUS IN MATHEMATICS

We have studied the syllabuses of several universities in India. Having regard to the comments we have made and the general principles laid down in the body of this report, we suggest the following syllabuses for the guidance of the mathematics boards of our universities, with a view to modernising their courses. The universities are, of course, free to modify them to suit their special requirements. The syllabus should be reviewed periodically and revised once in every five years, as suggested by the previous committee. We would like, however, to emphasize the need to change the pre-university and the undergraduate courses, as they provide the basic foundations.

## Pre-University (or Standard XI) Courses

Algebra one paper and Geometry and Trigonometry one paper. The time required for teaching the course is one hour a day roughly and not less than 160 hours during the year for covering the ground for both the papers.

### Paper I—ALGEBRA

Elementary theory of sets : Union, Intersection, Complimentation of a set. Venn Diagrams.

The number systems : The set of Natural numbers, zero as a number, Integers, Rational numbers, Real and Complex numbers. The fundamental operations with them and basic Laws of the rational and real numbers viz. Closure, Commutative, Associative, Distributive, Identity elements, inverse cancellation.

Indices, surds and logarithms, theory of quadratic equations. Relations, functions and their graphs, simple treatment of A.P., G.P., H.P., and also of A.M., G.M., and H.M., and the relations between them. Sum to infinity of geometric series. Mathematical Induction. The values of $\Sigma n$ and $\Sigma n^2$. Permutation and combination. Elementary course in Theory of Probability.

Binomial theorem for a positive integral index. Partial fractions, simple properties of determinants of second and third order. Solution of simultaneous linear equations by determinants.

### Paper II—GEOMETRY AND TRIGONOMETRY

*Pure Geometry : (Plane and Solid)*

(I) Basic concepts, important definitions and postulates in Geometry. Line, Plane, Angle between two skew lines, Angle between a line and a plane and angle between two planes. Dihedral Angle.

Tetrahedron, Pyramid, Cone Cylinder (general discussion only).

(II) Similar triangles, mean proportionals. To find a third and a fourth mean proportional. Important theorems on similar triangles.

Triangular prism. (No formal proofs for the areas and volumes are required).

The proofs of the following theorems only may be required in Solid Geometry :

1. If a straight line is perpendicular to each of two intersecting straight lines at the point of intersection, it is also perpendicular to the plane in which they lie.
2. All straight lines drawn perpendicular to a given straight line at a point on it are coplanar.
3. If two intersecting straight lines are respectively parallel to two other intersecting straight lines not in the same plane with them, the first pair and the second pair contain equal angles.
4. If two straight lines neither of which intersect nor are parallel, then (i) there is one straight line perpendicular to both of them, and (ii) this common perpendicular is the shortest distance between the given lines.
5. The lines joining the middle points of opposite sides of a tetrahedron are concurrent and bisect one another.
6. The lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent at a point which divides each of these lines in the ratio 3 : 1.
7. Any plane section of a sphere is a circle.

*Analytical Geometry :*

(III) Rectangular Co-ordinates, section formulae.

Straight line : Standard forms of its equation.

Conditions for parallel and perpendicular lines, Angle between two lines, bisectors of Angles, distance of a point from a straight line. Simple Locus problems.

*Circle :*

Equation of the circle, tangent, condition that a straight line be tangent to a circle, orthogonal circles, conditions for orthogonality, radical axis.

*Trigonometry :*

(IV) Angle as being generated by the rotation of a ray. Radiant measure. Trigonometrical ratios and their signs. Trigonometrical ratios of $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and of any angle between 0 and $2\pi$. Simple identities. Trigonometrical functions of $\left(\frac{\pi}{2}-\theta\right)$ $\left(\frac{\pi}{2}+\theta\right)$, $\pi-\theta$ etc.

Solution of right angled triangles and simple problems on heights and distances. Sine and Cosine Laws and geometric interpretation of the Cosine Law. Solution of triangles. Addition and multiple angle formulae. Simple identities. Simple equations involving trigonometric functions. Periodicity and graphs of sin $\theta$, cos$\theta$ and tan $\theta$.

Note: It may be possible for some universities to delete some portions of Geometry and Trigonometry and include in their place an elementary course in Calculus.

## B.A. and B.Sc. Part I (or Intermediate or the First Year Mathematics Course in Engineering)

The time required is roughly one hour a day or a minimum of 150 hours during the year.

### Paper I—CALCULUS AND GEOMETRY

*Differential Calculus :*

Functions, the Concepts of Limit and Continuity, Derivatives of Algebraic, Trigonometric, Exponential and Logarithmic Functions, Differentiation of Composite, Inverse and Implicitly defined functions and Functions defined parametrically. Successive Differentiation, Leibnitz's Rule, Tangents and Normals. Maxima and Minima of functions of a single variable.

*Integral Calculus :*

Integration of standard forms, Integration by partial Fractions, by substitution and by parts, Reduction Formulae, Evaluation of simple Definite Integrals, Definite Integral as an area and as the limit of a sum. Applications to area, Lengths of curves, volume and surface of a solid revolution.

*Analytic Geometry of two Dimensions :*

Standard Equations of Parabola, Ellipse and Hyperbola and their simple properties.

*Rigid Motion :*

The general idea of rigid motion, congruence of geometrical figures under rigid motion. Geometrical transformations of plane figures : (i) translations, (ii) reflections, and (iii) rotations under rigid motion.

### Paper II—ALGEBRA AND TRIGONOMETRY

Sets, Real numbers—Postulates and consequences. The ordering of the real numbers, the ordered field, the Basic Inequality Postulates—the trichotomy, the transitive, the addition and multiplication of inequalities. Simple Inequalities, Absolute values. Relations, functions and mappings. Complex numbers, Inequality.

Division or Elclid's Algorithm; Polynomials : Synthetic Division, Rational Roots and the Fundamental Theorem of Algebra. Relations between co-efficients and roots of a Polynomial equation. Graphing of Polynomial functions, quadratic, cubic and quartic curves.

Simple treatment of Matrices and determinants. Addition and Multiplication of Matrices. Rank of a Matrix. Solution of a system of linear equations involving not more than 4 variables.

Infinite series—limit of an infinite sequence, convergence of an infinite series—the comparison tests, the ratio tests, Cauchy's root test, alternating series. A monotonic sequence. Power series-convergence of Binomial and exponential logarithmic series.

Trigonometric functions, Inverse functions, Principal values. Complex numbers—geometrical representation of a complex number, De Moivre's theorem. The nth roots of unity. Series for Sine and Cosine, Hyperbolic functions, Logrithms of a complex number, summation of trigonometric series (simple cases only).

## Syllabi for B.A. and B.Sc. Parts II and III

We have studied the syllabuses adopted by some of our universities for the second and third years of the three-year degree course in mathematics. Some universities have an honours (major or principal) course and provide eight or six papers in Pure and Applied Mathematics. We are submitting below an eight-paper scheme for the honours course of which four papers are to be taken at the end of the second year and four others at the end of the third year. Some provide for the study of mathematics as one of the three different subjects taught in arts or science, while some provide for it as a pass course or as a minor subject. In these cases, there are usually three or four papers to be taken at the end of the two years. In the scheme given below a four-paper scheme is given of which two are to be taken at the end of the second year and the two others at the end of the third year. In view of the vast importance of Calculus (or Analysis) and Modern Algebra for all students no matter whether their interests lie in physics, engineering or economics, we consider that papers on these subjects should be common to all. The students who offer mathematics even as a minor or pass course could with advantage take Papers I and II at the end of the second year and papers IV or V, and VII of the honours course at the end of the third year.

The students of honours or major mathematics should take all the four papers I to IV at the end of the second year and the papers V to VIII at the end of the third year of the three-year degree course. In some universities, honours students have to take other subjects as well and may have time for only three papers each year in mathematics. In such cases, they may choose any subject they like from the remaining two papers (i. e. Papers III and IV and Papers V and VIII). Since we have recommended that there should be an examination of the university at the end of every academic year (if not at the end of every semester or term), we have divided the course into two parts—one for each year.

We have not suggested any text-book as there are many available. The teacher should choose them carefully. Most of these books are, however, expensive foreign books. We suggest that the UGC should get cheap books written by competent scholars, perhaps, with a subsidy for production,

## B.A./B. Sc. (Part II)

### Paper I—ALGEBRA AND GEOMETRY

*Mathematical Logic :*

(a) Notion of logical inference. Rules of inference and proof. Truth and validity—truth values and truth functional connectives.

Tautology—Predicates and universal quantifier.

*Modern Algebra :*

(b) Elementary properties of Integers, starting with Peano's axioms. The Modulo system. Euler's function. Fermat's, Wilson's and Lagrange's theorems. Integral domains. Notions of groups, rings, fields and vector spaces.

*Geometry* :

(c) General equation of a conic section and its reduction to standard forms. Polar coordinates. Equation of a conic section and its tangent in polar coordinates. Equation of a plane and a straight line. Standard equation of a sphere, cylinder, ellipsoid, paraboloid and hyperboloid and general properties of central conicoids.

(d) Elementary treatment of finite differences and interpolation formulae.

### Paper II—CALCULUS (ANALYSIS)

(a) The principle of the point of accumulation. Limits of sequences. Cauchy's convergence test. Monotonic sequences. Upper and lower bounds of a set. Upper and lower limits. Functions of a real variable. Continuity of functions. Properties of continuous functions.

(b) Derivatives of algebraic, trigonometric, exponential and logarithmic functions and inverse functions. Higher derivatives and Leibnitz's Theorem. Properties of derivatives. Rolle's theorem and Mean value theorem. Taylor's and Maclaurin's series. Maxima and minima. Indeterminate forms.

(c) The definite integral as the limiting value of upper and lower sums (Riemann Integral). Fundamental rules of operation. Indefinite integral, the derivatives of the indefinite integral. The fundamental theorem of the integral calculus. The mean value theorems of the integral calculus.

Reduction formulae. Integration of rational functions involving trigonometric and hyperbolic functions. Evaluation of definite integrals.

*Differential equations :*

(d) Differential equations of the first order. Exact differential equations. Simple equations of the second order.

*Applications :*

(e) Length of an arc of a plane curve. Concavity and convexity. Points of inflexion. Asymptotes. Surface area. Volumes of solids or revolution. Particle falling freely under gravity including air resistance. Simple harmonic motion.

## Paper III—PROBABILITY AND ELEMENTARY STATISTICS

(a) Frequency, definition of probability. Axiomatic approach to probability. Fundamental concepts of experiment, outcome, sample space (discrete case only), events and probability of events. Union (finite), intersection and complementation of events. Conditional probability, Bayes' Theorem. Independent events.

(b) Discrete random variable defined as functions on discrete sample spaces. Intuitive treatment of continuous random variable. Distribution function and probability density function. Expectation, moments, characteristic functions of measures of location, dispersion, skewness and kurtosis. Standard distributions—Binomial, Poisson, Hypergeometric, Negative Binomial, Uniform, Normal, Beta, Gamma and Cauchy distributions.

(c) Distributions of more than one variable. Conditional and marginal distributions. Dependent and independent random variables. Conditional expectation, Correlation and regression. Bivariate normal distribution.

(d) Tchebysheff's Lemma. Convergence probability. Weak law of large numbers. Statement and applications of central limit theorem. Poisson and normal limits of the Binomial distribution.

(e) Chi-square, t and f distributions. Sampling distributions of mean and variance in samples from a normal population. Standard errors of statistics. Large-sample standard errors.

(f) Elements of statistical inference—Unbiassedness, efficiency and consistency of estimaters. Method of maximum likelihood. Testing of simple hypotheses. Confidence intervals.

## Paper IV—VECTOR ANALYSIS AND MECHANICS

*Vector Analysis :*

(a) Vectors and Scalars. Rules of vector algebra. Vectors as an ordered triad of three real numbers. Scalar and vector Products. Vector equations. Independent and dependent vectors.

*Dynamics :*

(b) Newton's laws of motion. Equations of motion of a particle. Motion in a straight line. Work, energy and power. Motion of a system of particles. Laws of conservation of energy, linear momentum and angular momentum.

Impulsive forces. Tangential and normal components of acceleration. Radial and transverse components of acceleration.

*Statics :*

(c) Equilibrium of a particle and a rigid body under coplanar forces. Principle of virtual work. Centre of mass. Friction. Equilibrium of freely hanging strings and strings on rough plane curves.

*Hydrostatics :*

(d) Fluid pressure, Centre of pressure. Floating bodies and metacentre. Gas laws and atmospheric pressure.

## B.A./B.Sc. Part III—(Final Year)

### Paper V—SET THEORY, TOPOLOGY AND INFINITE SERIES

*Set Theory :*

(a) Basic notion of set theory. The concepts of a set and its subsets and set algebra. The cartesian product of two sets and definition of a general function. One to one mappings and their inverses. Composite functions. Sequences. The number of elements in a set. Countable sets. The set of all real numbers is uncountable. The notion of a cardinal number.

*Topology :*

(b) Notion of a topological space. Open sets, closed sets, derived sets. Elosure, neighbourhood, limiting point. Metric spaces. The real number system and its topological properties. Bolzano Weierstrass theorem. Continuous mappings, homeomorphism, compact and connected spaces. Application of compact and connected spaces in real analysis. Heine Borel Theorem.

*Infinite Series :*

(c) Tests of convergence including comparision, d'Alembert's, Cauchy's integral tests. Dirichlet's theorem on rearrangement of terms of an absolutely convergent series. Multiplication of series. Uniform convergence. Power series.

### Paper VI—CALCULUS (ANALYSIS)

(a) Derivatives in polar coordinates. Vector functions and their derivatives.

(b) Functions of two or more variables. Limits, continuity and differentiability of functions of more than one variable. Partial derivatives of a function of more than one variable. Total differential. Jacobians. Implicit functions and their differentiation. Maxima and minima of functions of two or more variables. Lagrange multipliers.

(c) Improper integrals. Infinite integrals.

(d) Differentiation and integration under the integral sign. Term by term differentiation and integration of series.

(e) Multiple integrals. Evaluation of simple, double and triple integrals. Beta and Gamma functions.

(f) Osculating circle and the circle of curvature. Involutes and Evolutes. Envelopes and singular points.

(g) Elements of Numerical integration.

### Paper VII—DIFFERENTIAL EQUATIONS, LINEAR ALGEBRA AND COMPLEX VARIABLES

*Differential Equations :*

(a) Singular solution. Standard linear differential equations with variable co-efficients. Simultaneous differential equations. The equation $P\,dx+Q\,dy+R\,dz=0$. Partial differential equations of the first order.

*Linear Algebra :*

(b) Matrices and determinants and their application to solutions of linear equations. Quadratic forms and their reduction to normal forms.

*Complex Variables* :

(c) Analytical functions. Cauchy-Riemann equations. Cauchy's theorem. Taylor and Laurent series. Residue theorem and contour integration.

### Paper VIII—ELECTRICITY AND MAGNETISM OR SPHERICAL ASTRONOMY

*Electricity and Magnetism* :

Inverse square law. Potential. Gauss's theorem. Electrostatic field. Lines and tubes of force. Equipotential surfaces. Conductors and insulators. Mechanical force on a conductor. Solution of simple electrostatic problems, method of images, boundary conditions. Condensers, dielectric constant, condensers in series and in parallel, capacity of a parallel plate condenser and of a sphere. Energy of charged bodies.

Steady currents. Ohm's law. Distribution of steady currents in closed circuits, Wheatstones' bridge.

Field of a magnetic doublet. Magnetic induction. Bar magnets, magnetic susceptibility and permeability. Magnetic field due to a steady electric current. Uniform magnetic shell. Forces between currents and magnets. Induced magnetism, Electromagnetic induction.

*OR*

*Spherical Astronomy* :

Fundamental formulae of spherical trigonometry and solution of spherical triangles (only as much of spherical trigonometry will be required as has a direct bearing on problems in spherical astronomy).

The celestial sphere and different systems of celestial co-ordinates. Twilight atmospheric refraction. Planetary motions and phenomena. Sidereal time, mean time, conversion of time. Equation of time. Aberration. Parallaxes of the Moon and the Sun and annual parallax of stars. Precession and Nutation. Elementary theory of eclipses.

## M.A./M.Sc. PART I

### Paper I—REAL ANALYSIS

(a) Dedekind and Cantor theories of irrational numbers. Continuous functions of one and more than one variable. Functions of bounded variation and their properties. Partial derivatives and partial differentials. Total differential. Partial differential derivatives of the second order. Young's and Schwarz's theorems. Implicit functions. Young's existence theorems.

(b) Fundamentals of set theory. Concept of cardinal numbers. General concept of function. Measure of plane sets. Measure on semi-rings. Extension of Jordan measure. Complete additivity. General problem of extension of measures. Lebesgue's extension of a measure defined on a semi-ring with unity. Extension of Lebesgue measure in the general case. Measurable functions and their fundamental properties. Sequences of measurable functions and their convergence under different concepts of convergence. Lebesgue integral of simple functions. The general definitions and fundamental properties of the Lebesgue integral. Passage to the limit under the Lebesgue integral. Comparison of Lebesgue and Riemann integrals. Products of sets and measures. Fubini's theorem. The integral as a set function. Riemann-Stieltje's and Lebesgue-Stieltje's integrals and their properties.

Square integrable functions. Mean convergence. Two spaces with countable bases. Orthogonal sets of functions. Orthogonalisation. Fourier series over orthogonal sets. The Riesz-Fischer theorem.

## Paper II—MODERN ALGEBRA

Semi-groups and groups. Cyclic groups. Coset decomposition of a group. Homeomorphism and Isomorphisms of groups.

Rings, sub-rings, ideals, Homeomorphism and Isomorphisms of rings. Anti-isomorphism. Characteristic of ring. Algebra of subgroups of the additive groups of a ring.

Extensions of rings and fields. Polynomial rings. Structure of any field. The number of roots of a polynomial in a field. Symmetric polynomials. Rings of functions. Elementary factorisation theory. Gaussian and Euclidean domains.

Groups with operators, M-Groups, Schreier's refinement theorem. The Jordan-Holder theorem. Fields : separable and inseparable. Algebraic extensions. Transcendental extensions. Perfect fields. Galois fields.

## Paper III—MATHEMATICAL METHODS

Avanced Vector analysis. Gradiant, divergence, curl and Laplacian. Theorems of Gauss, Green and Stokes. Serret-Frenet formulae. Curvature and torsion of curves. Cartesian Tensors. Covariant and contravariant tensors. Equation of a geodesic in a Riemannian space. Ordinary differential equation. Self adjoint operators, eigen-value problem for the Sturm-Liouville equation. Orthogonality. Eigen-function expansion. Partial differential equations. Laplace's equation, transformation of Laplace's equation from rectangular coordinates to curvilinear coordinates. The maximum minimum property and the fundamental properties of harmonic functions. Euler's problem in the Calculus of variations. Fourier and Laplace transforms.

## Paper IV—FUNCTIONS OF A COMPLEX VARIABLE

Analytic functions. Liouville's theorem. Morera's theorem. Singularities of analytic functions. Behaviour of an analytic function in the neighbourhood of a singularity. Rouche's Theorem. Argument principle. Mittag-Leffler's theorem. Maximum modulus theorem. Hadamard's three circle theorem. Schwarz's Lemma. Analytic continuation. Monodromy theorem. General analytic function. Power series and the formula for its radius of convergence. General tests for singular points. Convergence of the series and regularity of the functions. Abel's theorem and its converse. Tauber's theorem and Littlewood's theorem. Functions with natural boundaries. Conformal mapping, linear transformations, normal families. Riemann mapping theorem. Schwarz-Christoffel formula.

## Paper V—TOPOLOGY AND FUNCTIONAL ANALYSIS

Sets : Set operations, Product sets, denumerable and non-denumerable sets, Equivalence of sets. Cardinal number. Equivalence relation. Mappings. Partially ordered sets, Zorn's lemma (without proof).

Metric spaces : Definition and examples. Neighbourhoods. Convergence, limit points and closure. Open and closed sets. Separable metric spaces. Continuous mappings. Homeomorphism. Isometry. Complete metric spaces. Sets of the first and second category. Baire's theorem. Diameter of a set. Bounded and totally bounded sets. Sequential compactness. Compact and locally compact sets. Connected sets. Principle of contraction mapping in a metric space.

Existence theorems for Differential Equations. Existence theorems for Integral Equations. Arzela's theorem and its applications. Generalised Arzela's theorem. Real function in a metric space. Extension of continuous functions from closed sets. Continuous curves in a metric space.

Normed Linear Spaces : Definition. Examples. Convex sets. Linear functionals. Confugerate space. Benach spaces. Hahn Banach's theorem. Closed graph theorem. Weak and strong convergence. Linear operators. Bounded operators. Inverse and Adjoint operators. Hilbert spaces. Schwarz's inequality. Orthogonal complements. Riesz-representation theorem. Separable Hilbert spaces. Orthnormal systems. Fourier coefficients. Bessel's inequality. Parseval's equality. Riesz-Fisher theorem.

Definition and examples of generalised functions (distributions) on the real line. Dirac's function. Derivative and convergence of generalised functions.

## MA./M.Sc.—PART II

Here we do not want to restrict the teacher's freedom of choice of topics. Hence we mainly indicate the subjects to enable the teachers to carry out as much advanced work as possible in these subjects.

### Pure Mathematics

Paper VI—SPECIAL FUNCTIONS AND INTEGRAL TRANSFORMS.

Paper VII—ADVANCED REAL ANALYSIS.

Paper VIII—ALGEBRAIC GEOMETRY OR DIFFERENTIAL GEOMETRY INCLUDING RIEMANNIAN GEOMETRY.

Two papers IX and X may be chosen from Schedule A and/or from Schedule B.

*Or*

### Applied Mathematics

Paper VI—ANALYTICAL DYNAMICS AND GRAVITATION.

Paper VII—MECHANICS OF CONTINUOUS MEDIA.

Paper VIII—ELECTROMAGENTIC THEORY AND RELATIVITY. TWO PAPERS IX AND X MAY BE CHOSEN FROM SCHEDULE B.

### Schedule A—Pure Mathematics

Set Theory. Foundations of Mathematics. Mathematical Logic. Algebraic Theories. Theory of Groups. Hilbert spaces. Combinatorial Topology. Probability, Theory (Advanced), Stochastic processes. Infinite Matrices and Summability. Linear Operators. Information Theory. Integral Equations. Theory of distributions. Econometrics. Sequences and function spaces. Theory of numbers. Harmonic Analysis.

### Schedule B—Applied Mathematics

Hydrodynamics. Aerodynamics. Elasticity and Plasticity. Quantum Mechanics. Electromagnetic Theory. Theory of waves and vibrations. Partial differential equations of mathematical physics, Non-Linear analysis, Stellar structure, Relativity, Classical Mechanics, Ballistics, Information Theory, Cybernetics, Special Functions, Celestial mechanics and satellite motion, Statistical mechanics and Thermodynamics, Turbulence, Magnetohydrodynamics.

Note : Subjects like Probability, Statistical Inference, Numerical Analysis, Stochastic processes can be classified as either Pure Mathematics special subjects or Applied Mathematics special subjects.

Finally, we thank the University Grants Commission for appointing this committee to review the position of mathematics in our universities and we must say that we have enjoyed this work and done our best to give a workable solution to the problem of modernising the teaching of mathematics at the undergraduate as well as at the postgraduate classes. Whether this report will serve any useful purpose or not will depend on the seriousness with which the teachers of mathematics view the urgency of the problem. We may venture to suggest that the University Grants Commission can also effectively bring to the notice of the universities the desirability of improving the courses in mathematics. Whether any university does real valuable work in mathematics can to a great extent be judged by the modernisation of their courses and development of progressive schools of research. The U.G.C. is the custodian of university interests and it is open to the Commission to see whether the funds advanced by them have been actually put to good use or not. Even the sanction of further development schemes should depend on the earnestness of the universities to improve the quality of mathematical education. Some people may complain that the number of papers has been increased to ten from eight for M.A. or M.Sc. examination. In this connection, we would like to remind them that in Mathematics Tripos they set *six* papers each year.

As pointed out by Academician Alexandrov, in a programme for improvement of mathematics teaching, computational mathematics is very much to be emphasised. It is, therefore, necessary that atleast some selected universities should be encouraged to have computors to enable them to teach computational mathematics. We suggest that at least 10 universities be provided with computors during the Fourth Five Year Plan.

# APPENDIX

## Record of Discussions with Academician Prof. A. Alexandrov, Rector, Leningrad University, regarding Revision of Mathematics Course

The aim of education is not just to acquire knowledge. It is to lead to creativity on the part of the learners. To bring about creativity exercises have to be given to the students. The students have to be assigned some topics for study and to present small papers. Sufficient opportunities have thus to be given to students to solve problems. The problems to be solved may not only relate to one hour duration but may take even a month. Not only to know but also to be able to apply and to create is an important thing. Competitions should be arranged at different levels. Starting with competitions at small regional levels one may go on making successive selections and finally arrange a competition even at the national level.

Examinations are intended not only to know what the students know but also what the students can invent and create. The students have to be inspired to have inventive ideas.

It is not enough to just write good syllabus. We require teachers who are themselves interested in creating and who can inspire.

There has recently been a revolutionary shift of emphasis as to the subjects to be taught and the manner they are to be taught. Computational mathematics needs to be very much emphasised. Moreover, computational mathematics does not have to be taught only from a practical point of view, but also theoretical aspects of the subject have to be borne in mind. Thus the formulation of a solution to a problem in a closed form is not of much importance. The solution has to be presented in the form of algorithms so as to be amenable to computation. Considerable stress has to be laid on connections between pure mathematics and applications and possibilities of adaptation to competition have to be constantly borne in mind.

The study of mathematical logic is very important not only from a theoretical point of view but also because of its connections with computing methods. A solution to any problem has to be broken up into a number of elementary stages.

Foundations of Geometry need to be taught specially when study of mathematical logic has to be emphasised. No separate study of Analytical Geometry is necessary in as much as the various properties such as focal properties are not of any use. Geometry need not be taught separately from Linear Algebra. We should have theory of curves in vectorial presentation. Complex functional theory must be taught as a compulsory course. But it may, however, be taught in smaller portions. While the Picard theorem is a marvellous result it is not necessary to do it. As a result of omitting some topics we may be able to find time for more important mathematics. Study of partial differential equations has to be heavily emphasised not only from the classical analysis point of view but also from the modern point of view of Functional Analysis. Thus the study of partial differential equations has to be up-to-date and more modern.

A study of set theory which is important must include the theory of transfinite numbers with special reference to enumerability. If it could be found possible to teach the Lebesgue theory of integration in an intelligent manner to any batch of students, Riemann theory may not be taught to that batch.

Special efforts have to be made to locate talented students and to provide for their special training in special institutions.

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